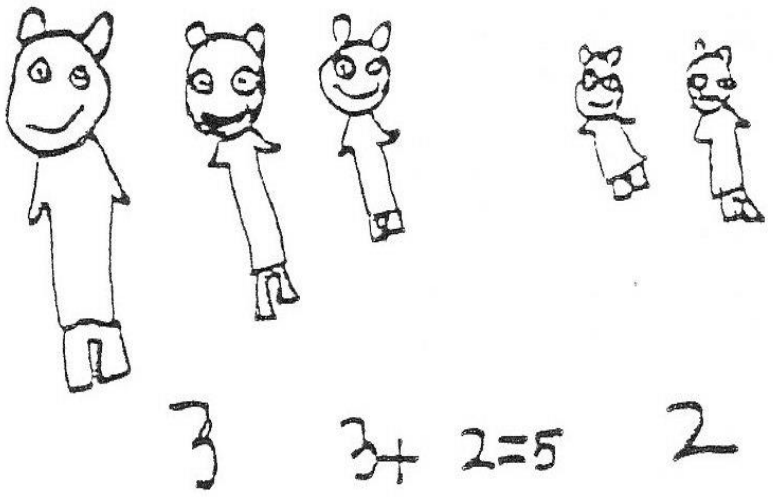
Dawlish Learning Partnership
Exminster Primary School

Guide to Written Calculations



Written Calculations

The purpose of this booklet is to outline the development and progression of **written calculations** your child will meet as they work their way through our school. We hope that it will help you towards a better understanding of the written methods that your child will be learning.

What are written calculations?

Mental calculation – the ability to calculate 'in your head' – is an important part of daily life – adding items of shopping, working out discounts etc. But as calculations become more complex, written methods become more important. They help us to keep track of our thinking and to work out calculations which are too difficult to do wholly mentally.

When do children learn to write out their calculations formally?

It is likely that when we were at school we were taught formal ways to set out our calculations. Children are still taught these 'traditional' methods but only when they fully understand what they are doing with the hundreds, tens and units. We firmly believe that children need to know what is happening to the numbers that they are dealing with, and not just to learn a written method off by heart.

The National Numeracy Strategy Framework for Teaching Mathematics (NNS) makes it clear that by the end of Year 6 all children will understand, and use successfully, conventional written methods to carry out and record calculations they cannot do 'in their heads'.

To get to this point, children in the Early Years do lots of practical work and mental maths involving number and counting. They develop a firm understanding and use of mental calculation strategies which continues throughout the school.

Alongside the teaching of mental calculation, we begin to teach written methods. We build on what children know, understand and can do, teaching written methods which emphasise what is happening to the numbers in the calculation.

The following methods of recording are recommended by the NNS, to be taught alongside children's growing understanding and use of mental calculation.

We hope you find this helpful. Please come and see us if you have any further queries.

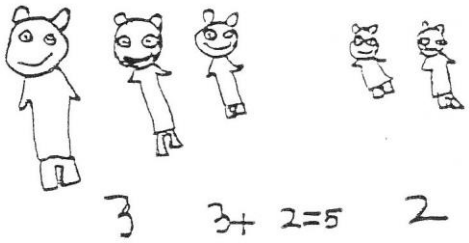
ADDITION

PROGRESSION THROUGH CALCULATIONS FOR ADDITION

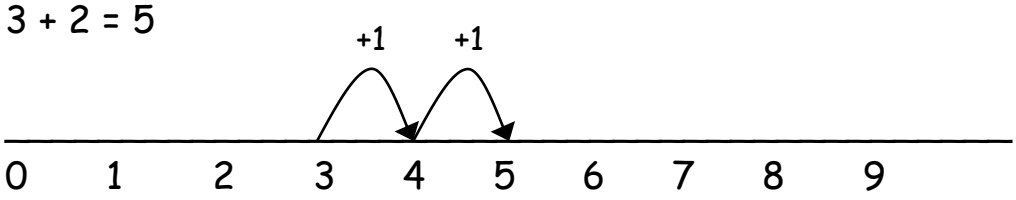
First Stage

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They will begin by drawing simple pictures to represent what they have done practically.

They will learn about and write the symbols + = involved in addition and also learn a wide range of vocabulary to talk about what they are doing such as *plus, add, count on, equals, totals, makes, altogether.*



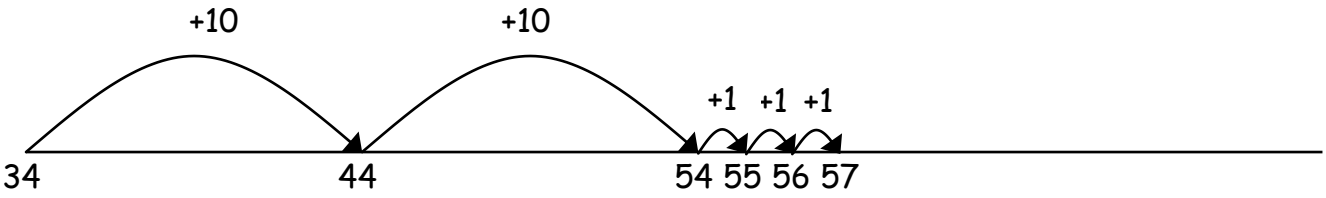
Children use number lines and practical resources to support calculation and teachers *demonstrate* the use of the number line. Children need to be taught that e.g. to add 3 and 2 they must start at three on the number line and make two jumps of one forward.



Second Stage

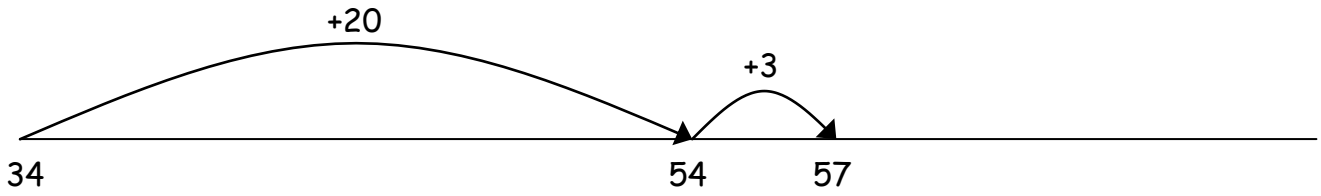
Children will begin to use 'empty number lines' to support their counting and addition. They draw their own blank line and write the numbers needed onto it.

34 + 23 = 57



When ready, children become more efficient at using the empty number line by adding the units in one jump and also by adding the tens in one jump. To be able to do this, children need to know simple number facts such as 30+20 and 4+3.

34 + 23 = 57



Third Stage

Children begin to tackle the calculation by adding the tens and then the units. We call this *partitioning:*

67+24=

60 + 7
+ 20 + 4
80 +11 = 91

Calculation involving 3 digits can be done in a similar way.
Children add the units, then the tens, then the hundreds.

167+124=

$$\begin{array}{r} 100 + 60 + 7 \\ +\ 100 + 20 + 4 \\ \hline 200 \quad 80 \quad 11 \end{array} = 291$$

This leads to:

126 + 112 =

$$\begin{array}{r} 126 \\ +\ 112 \\ \hline 8 \\ 30 \\ 200 \\ \hline 238 \end{array}$$

213.5 + 112.3 =

$$\begin{array}{r} 213.5 \\ +\ 112.3 \\ \hline 0.8 \\ 5.0 \\ 20.0 \\ 300.0 \\ \hline 325.8 \end{array}$$

Fourth Stage

Finally, children learn the *column method:*

$$\begin{array}{r} 625 \\ +\ 48 \\ \hline 673 \\ {\scriptstyle 1} \end{array} \qquad \begin{array}{r} 783 \\ +\ 42 \\ \hline 825 \\ {\scriptstyle 1} \end{array} \qquad \begin{array}{r} 367 \\ +\ 85 \\ \hline 452 \\ {\scriptstyle 1\,1} \end{array}$$

Using similar methods, children will:

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;*
- ✓ *know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p*
- ✓ *begin to add two or more decimal fractions with up to three digits and the same number of decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m – 280 cm.*

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1) **they are not ready.**
2) **they are not confident.**

Children should be encouraged to estimate their answers before calculating.
Children should be encouraged to check their answers after calculation using an appropriate strategy.
Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

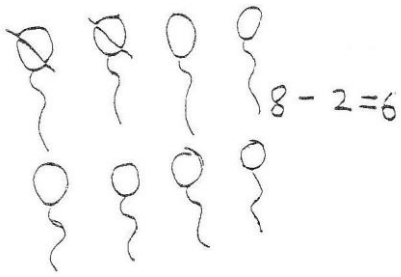
SUBTRACTION

PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

First Stage

Children need lots of practical activities of 'taking away'. They do plenty of counting forwards and backwards in 1's, 2's and 10's. They sing number rhymes and songs which involve subtraction such as 10 fat sausages and 5 little speckled frogs.

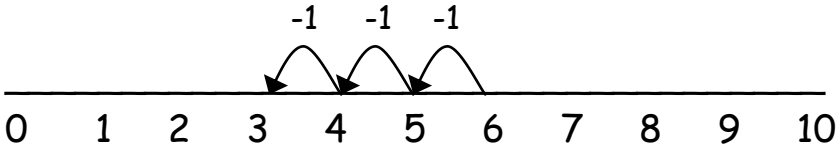
As with addition, children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They draw pictures to represent the calculation. They will learn about and write the symbols - = involved in subtraction and also learn a wide range of vocabulary to talk about what they are doing such as *minus, take away, subtract, count back, difference between, equals, totals, makes.*



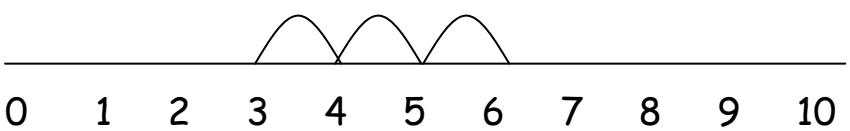
A shepherd looks after 8 sheep.
He has lost 5. Now he has 3 left.

Children use number lines and practical resources (such as counters) to support calculation. Teachers *demonstrate* the use of the numberline.

6 – 3 = 3



The numberline should also be used to show that 6 - 3 means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.



It is really important that children develop an awareness that subtraction is not always 'taking away'.

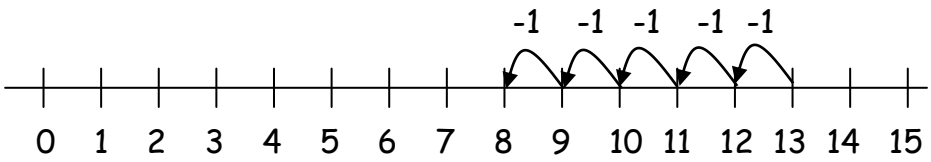
Jane has 6 sweets. She gives 3 sweets to Emma. How many sweets does Jane has left.

Jane has 6 sweets and Emma has 3 sweets. How many more sweets does Jane have than Emma.

Both of these situations can be written as 6 – 3 = 3 but one is 'taking away' and the other is 'finding the difference'.

Children then begin to use number lines to support their own calculations - using a number line to count back in ones.

13 – 5 = 8

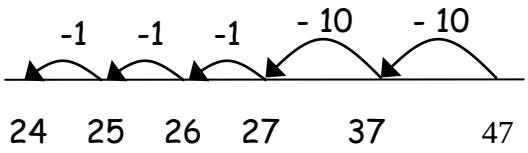


Second Stage

Counting back/partitioning

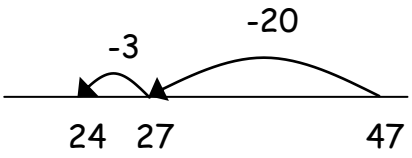
Children move on to using an empty number line to subtract. They draw a blank line and write the numbers needed for the calculation.

47 – 23 = 24



When ready, children become more efficient at using the empty number line by subtracting the units in one jump and also by subtracting the tens in one jump. To be able to do this they need to know simple number facts such as 40-20 and 7-3.

47 – 23 = 24



Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

For some children counting up can be confusing as they do not understand why they are 'adding' in order to 'take away'. This usually means that they do not have a clear understanding of subtraction as 'finding the difference' where nothing is taken away.

Third Stage

Decomposition

Children will continue to use empty number lines with increasingly large numbers. They also begin to write their calculations in a slightly more formal way, where the idea of tens and units lining up is introduced.

We call this *decomposition.*

89	=	80	+	9		
- 57		50	+	7		
		30	+	2	=	32

Initially, the children will be taught using examples that do not need them to 'exchange'.
'Exchanging' is necessary in the following example. A ten is exchanged for ten units. Children then deal with the units and then look at whether they can do the tens.

754 – 86 =

```
 614 1
 7 5 4
-  8 6
 6 6 8
```

567.3 – 378.6 =

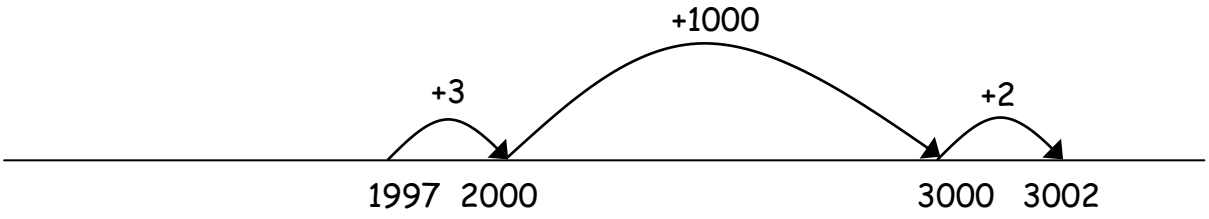
```
 4 15 6  1
 5 6 7 .3
- 3 7 8 .6
   1 8 8 .7
```

Children should:

- ✓ *be able to subtract numbers with different numbers of digits;*
- ✓ *be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;*
- ✓ *know that decimal points should line up under each other.*

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

3002 – 1997 = 1005



By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1) they are not ready.
2) they are not confident.

Children should be encouraged to estimate their answers before calculating.
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Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

MULTIPLICATION

PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

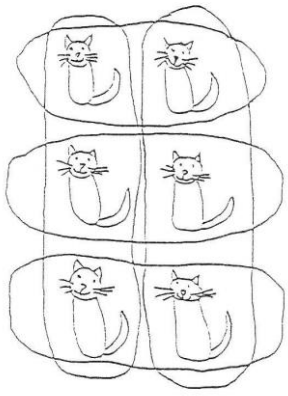
First Stage

An understanding of multiplication and division begins with rhymes and stories which involve counting (1,2, buckle my shoe…), counting forwards and backwards in 1's, 2's, 5's and 10's and doubling and halving numbers. All of these things help develop a familiarity with numbers and number sequences.

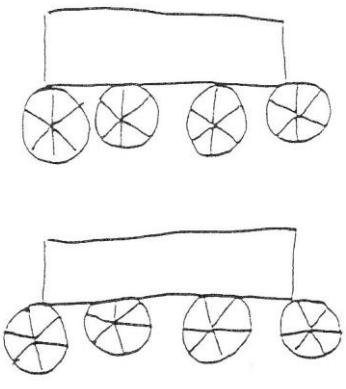
Alongside the counting goes practical activities using actual objects (15 cards shared between 3 players, 12 pencils shared between a group of 6 children). Children will work on practical problem solving activities involving equal sets or groups.

Visual images help children see how different numbers can be put together and split up. Discussion helps develop understanding and use of appropriate language. Practical activities, counting out loud and talking about 'sharing' and 'lots of' precede any formal written method.

Children draw rings around objects to show how they can be split.
This shows the cats as 2 lots of 3 or 3 lots of 2.



Children will draw pictures to record their thinking. They will be encouraged to talk about grouping 8 into 2 groups of 4.



How many cars can you make if you have 8 wheels?

Second Stage

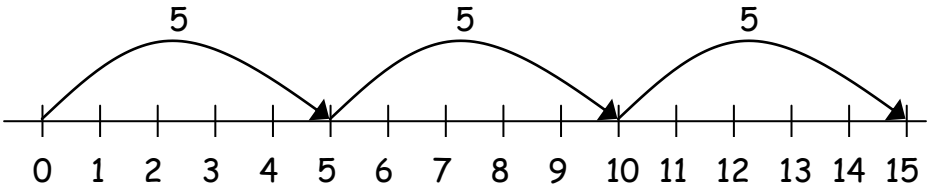
Children will develop their understanding of multiplication and use jottings to support calculation. They will be introduced to the symbols involved in multiplication x = and will also learn to use a wide range of vocabulary to help them talk about their calculations such as *times, multiplied by, lots of, makes, equals*.

Repeated addition

3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3

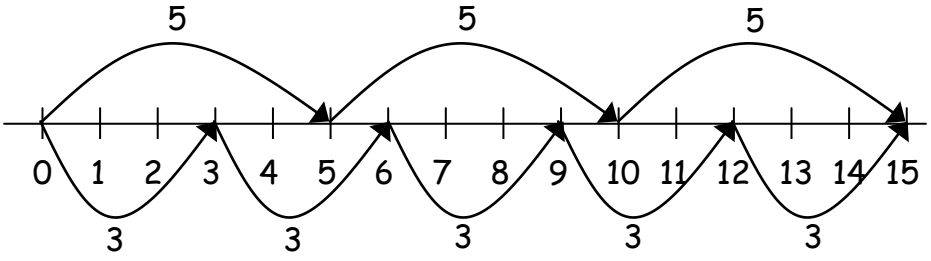
Repeated addition can be shown easily on a number line:

5 x 3 = 5 + 5 + 5



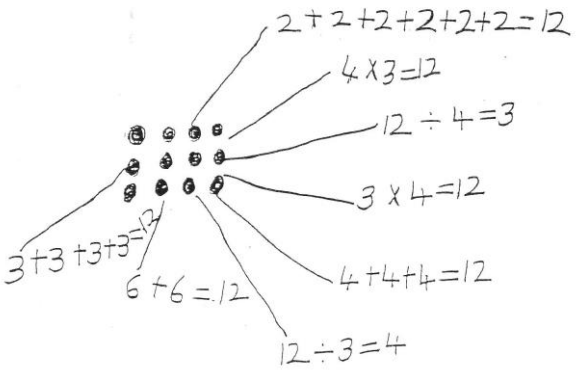
Commutativity

From an early stage we teach children to make use of known facts. For example if you know that 3x5 =15 you also know that 5x3 =15.
This can be shown on the number line.



Dotty Arrays

Children should be able to draw a multiplication calculation using an array. This knowledge and visual image will support with the development of the grid method. The use of this method will continue into Key Stage 2.

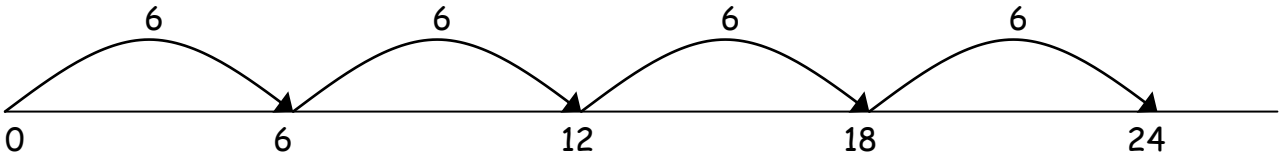


Third Stage

Repeated addition

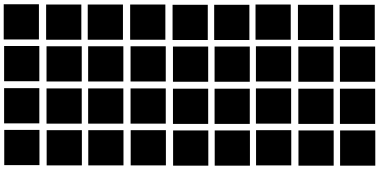
4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4

Children should use number lines to support their understanding.



Dotty Arrays

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



4 x 9 = 36

9 x 4 = 36

They will also learn to use symbols to stand for unknown numbers to complete equations using inverse operations. The importance of knowing number facts is evident here, for example, knowing that 4x5 is 20 or that 3x6 is 18. Similarly, being able to consider how many 5's are in 20, or to ask what is 20 divided by 5 enables children to solve these calculations.

□ x 5 = 20 3 x △ = 18 □ x O = 32

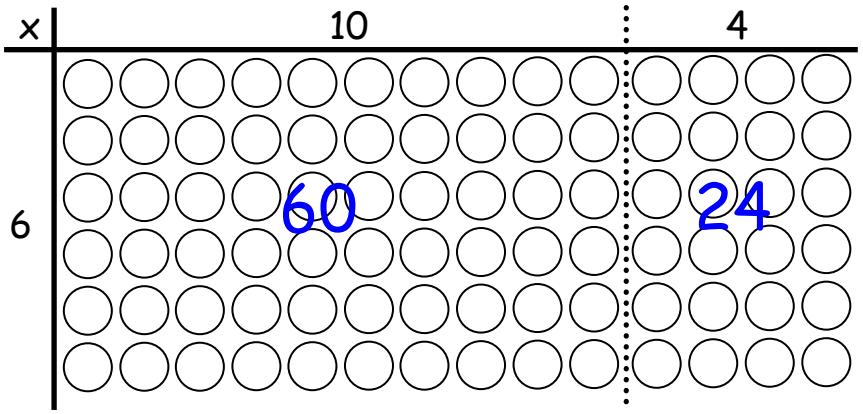
Partitioning

38 x 5 = (30 x 5) + (8 x 5)
= 150 + 40
= 190

Children will continue to make use of known multiplication facts to help break calculations down.

Fourth Stage

Children will continue to use arrays where appropriate leading into the grid method of multiplication. 6 x 14 =



(6 x 10) + (6 x 4)

60 + 24

84

Grid method

Children learn the grid method of multiplication.
23 x 8 (TU x U) is calculated in this way.
Children will estimate first.

23 x 8 is approximately 25 x 8 = 200

Step 1

x	20	3
8	160	24

Step 2

160
+ 24
184

72 x 38 (**TU x TU)**
Children will estimate first.
72 x 38 is approximately 70 x 40 = 2800

Step 1

x	70	2
30	2100	60
8	560	16

Step 2

2100
+ 560
+ 60
+ 16
2736
1

Using similar methods, children will be able to multiply decimals, approximating first. They should know that the decimal points line up under each other.
e.g. 4.9 x 3
4.9 x 3 is approximately 5 x 3 = 15

Step 1 We think of the 3 x 0.9 part of this calculation as 3 x 9, remembering then to divide 27 by 10 to revert the answer back to tenths.

x	4	0.9
3	12	2.7

Step 2

12
+ 2.7
14.7

4346 x 8 **(ThHTU x U)**
Children will estimate first.
4346 x 8 is approximately 4346 x 10 = 43460

Step 1

x	4000	300	40	6
8	32000	2400	320	48

Step 2

32000
+ 2400
+ 320
+ 48
34768

Using similar methods, children will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.

4.92 x 3
Children will estimate first.
4.92 x 3 is approximately 5 x 3 = 15

x	4	0.9	0.02
3	12	2.7	0.06

12
+ 0.7
+ 0.06
12.76

When the understanding is there, then children can be moved towards the more traditional approach. However, some children will choose not use this method. Some feel more secure using the grid method.

```
1 2 5
  5 x
  2 5 (5 x 5)
1 0 0 (20 x 5)
5 0 0 (100 x 5)
6 2 5
```

```
  7 2
  3 x
2 1 6
```

```
  3 8
  7 x
2 6 6
  5
```

```
  7 4 2
    8 x
5 9 3 6
  3 1
```

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

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DIVISION

PROGRESSION THROUGH CALCULATIONS FOR DIVISION

First Stage

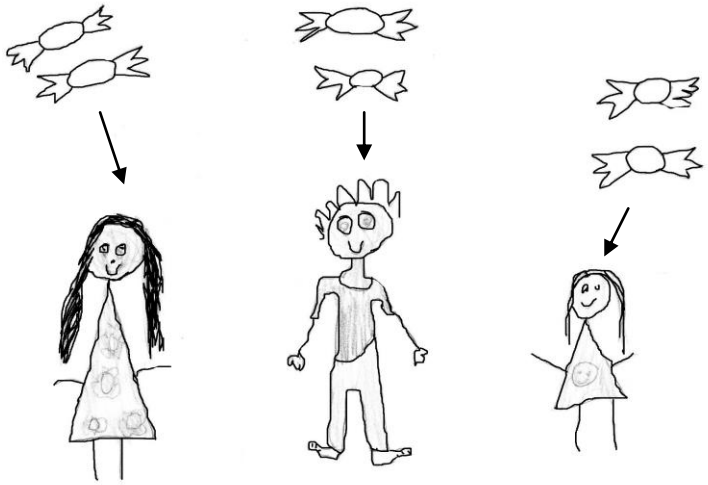
Children learn about equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

Second Stage

Children develop their understanding of division and draw pictures and use jottings to support calculation.
The division symbol will be introduced and children will also learn to use a wide range of vocabulary to help them talk about their calculations such as *divided by* and *shared between.*
Children continue to describe and develop their understanding of dotty arrays using ÷ and x symbols.

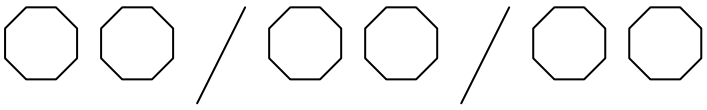
Sharing equally

6 sweets shared between 3 people, how many do they each get?



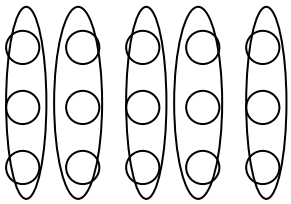
Grouping or repeated subtraction

There are 6 sweets, how many people can have 2 sweets each?

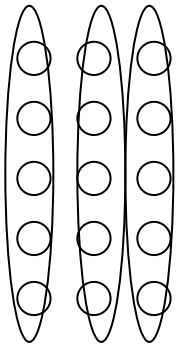


Dotty Arrays

Arrays should be used to develop children's understanding of the links between multiplication and division.



15 ÷ 3 = 5

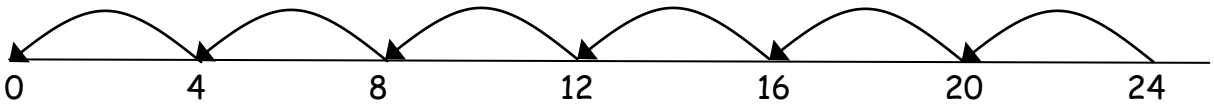


15 ÷ 5 = 3

Repeated subtraction using a number line

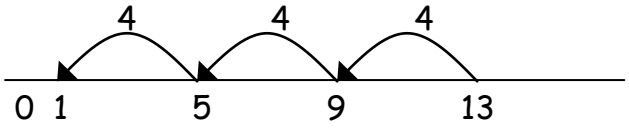
Children will use an empty number line to support their calculation.

24 ÷ 4 = 6



Children should also move onto calculations involving remainders.

13 ÷ 4 = 3 r 1 (We say this as 3 remainder 1)



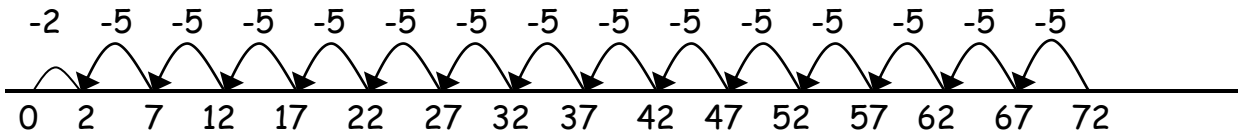
Children will learn to use symbols to stand for unknown numbers to complete equations using inverse operations. Knowing number facts and times tables is essential for this and all efficient calculating.

26 ÷ 2 = □ 24 ÷ △ = 12 □ ÷ 10 = 8 6 = □ ÷ △

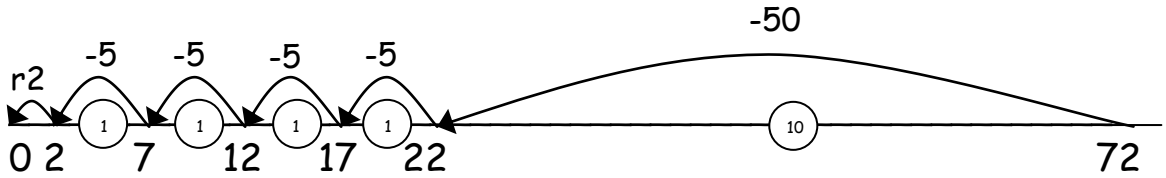
Third Stage

Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor (the number we are dividing by). Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the children are more familiar.

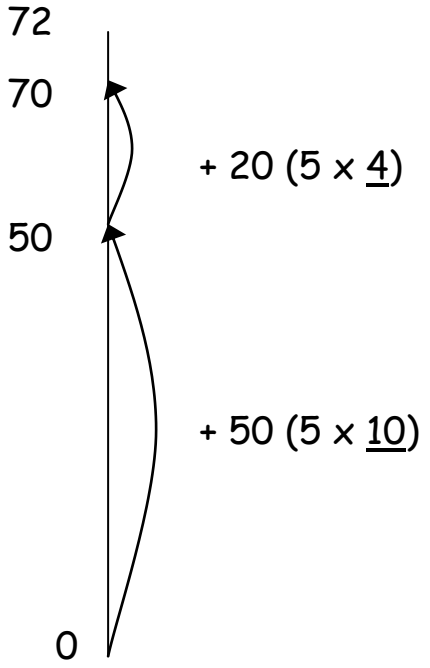
72 ÷ 5



Moving onto:



For some children – who maybe are less confident with subtraction – using a vertical number line to add on multiples of the divisor until the target number is reached is a useful method. 'How many 5s are there in 72?'



+ 20 (5 x 4)

+ 50 (5 x 10)

72 ÷ 5 = 14 r 2

Then onto the vertical method:

Short division TU ÷ U

Children move on to the following written method where division can be seen as 'breaking down' numbers. The 'breaking down' will often involve multiples of 10 because this is a multiple children will be familiar with.

A shop notice states that there are 87 shopping days to Christmas. How many weeks is that?

```
    12  r3
  ------
7 ) 87
  - 70      10 x 7
    17
  - 14       2 x 7
     3
```

Answer: 12 weeks and 3 days

Any remainders should be shown as integers, i.e. 12 remainder 3 or 12 r 3.

Children should make sensible decisions about rounding up or down after division. For example 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

e.g. I have 62p. Sweets are 8p each. How many can I buy?
Answer: 7 (the remaining 6p is not enough to buy another sweet)

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?
Answer: 8 (the remaining 6 apples still need to be placed into a box)

Fourth Stage

Children can start to subtract larger multiples of the divisor, e.g. 30x

Short division HTU ÷ U

196 ÷ 6

```
      32 r 4
   ------
6 ) 196
  - 180      30x
    16
  -  12      2x
     4
```

Answer : 32 remainder 4 or 32 r 4

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Fifth Stage

Children will continue to use written methods to solve short division TU ÷ U and HTU ÷ U.

Long division HTU ÷ TU

972 ÷ 36

```
      27
   ------
36 ) 972
   - 720      20x
     252
   - 252      7x
       0
```

Answer : 27

This leads onto

```
      32  r4
   ------
    1
6 ) 196
```

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1) **they are not ready.**
2) **they are not confident.**

Children should be encouraged to approximate their answers before calculating.
Children should be encouraged to check their answers after calculation using an appropriate strategy.
Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

The importance of knowing number facts

It is very important for children to learn their number bonds e.g. 7+3 = 10 and 3+7 = 10, and also corresponding subtraction facts e.g. 10-7 = 3 and 10 – 3 = 7. Knowing these leads onto knowing more number facts such as 17 + 3 = 20 and 20 – 3 = 17 and also facts such as 70 + 30 = 100 and 30 + 70 = 100 and so on.
Knowing times tables and corresponding division facts by heart is essential for efficient calculating and problem solving e.g. 7 x 8 = 56 and 56 ÷ 8 = 7.

The wider use of written methods in Years 5 and 6

Once written methods for addition, subtraction, multiplication and division have been learnt they can then be used in a range of problem solving activities involving fractions, decimals and percentages.

We hope that this information gives you a clearer idea of the written methods your child will meet as they progress through the school but please come and see us if you have any questions.